Lec. 4

Deviance

$$D = 2 \log \frac{\mathcal{L}(\theta_{best} \mid y)}{\mathcal{L}(\hat{\theta} \mid y)}$$

$$= 2\left(\ell(\theta_{best} \mid y) - \ell(\hat{\theta} \mid y) \right)$$

Poisson

$$\mathcal{L}(\lambda \mid y) = \prod \frac{\lambda^{y_i} e^{-\lambda}}{y_i!} \qquad \ell(\lambda \mid y) = \sum y_i \log \lambda - \lambda - \log y_i!$$

Best: $E(y_i) = y_i$ model: $E(y_i) = \hat{\lambda}$

$$D = 2\left(\sum \left(y_i \log y_i - y_i - \cancel{\log y_i!} \right) - \sum \left(y_i \log \hat{\lambda} - \hat{\lambda} - \cancel{\log y_i!} \right) \right)$$

$$= 2 \sum \left(y_i \log \frac{y_i}{\hat{\lambda}} - (y_i - \hat{\lambda}) \right)$$

Normal

$$\ell(\mu \mid y) = \sum_{i=1}^{n} \left(-\frac{1}{2} \log 2\pi\sigma^2 - \frac{1}{2\sigma^2} (y_i - \mu)^2 \right)$$

Best $\quad E(y_i) = y_i$
$\quad \hookrightarrow \mu$

Model $\quad E(y_i) = \hat{y}$
$\quad \hookrightarrow \mu$

$$D = 2\left(\sum_{i=1}^{n} \left(-\frac{1}{2} \cancel{\log 2\pi\sigma^2} - \frac{1}{2\sigma^2} (\underbrace{y_i - y_i}_{0})^2 \right) - \sum_{i=1}^{n} \left(-\frac{1}{2} \cancel{\log 2\pi\sigma^2} - \frac{1}{2\sigma^2} (y_i - \hat{y})^2 \right) \right)$$

$$= \sum_{i=1}^{n} \frac{(y_i - \hat{y})^2}{\sigma^2}$$